# Theory

## Background

In a cooling tower with open water circulation, heat is removed from water because of the material and heat exchange between the water and the ambient air. The cooling tower is a special form of heat exchanger because in addition to heat exchange, a material exchange also occurs as a result of evaporation. In fact, evaporation is the main mechanism by which the water is cooled in the cooling tower.

Evaporation is a process in which a liquid evaporates into the gas phase of another material because its vapor pressure in that gas phase is less than the saturated vapor pressure. This cools the liquid by removing the latent heat of evaporation from it. The cooling effect of ambient air is determined by its capacity to take up water vapor which is directly related to the air humidity. Cooling by evaporation takes place only if the relative air humidity is less than 100 %.

An additional mechanism of water cooling is convective heat transfer between air and water. This is a pure heat transfer without any kind of material exchange. The driving force for this mechanism is the temperature difference between water and air.

## Thermodynamic Properties of Humid Air

**Humidity** $h$ of an air-vapor mixture is defined as the mass ratio of water vapor and dry air,

$$h = \frac{m_W}{m_{DA}} = \frac{M_W N_W}{M_{DA} N_{DA}} \tag{2.1}$$

Here, indices $W$ and $DA$ refer to the water vapor and dry air, respectively, and $m_x$, $M_x$ and $N_x$ are the mass, molar mass, and the number of moles of $x$ ($x$ = $H_2O$ or $DA$). At typical operating conditions both dry air and water vapor can be approximated as an ideal gas, i.e.

$$N_x = \frac{P_x V}{RT}, \tag{2.2}$$

where $P_x$ is the partial pressure of $x$ ($x$ = $W$ or $DA$), $V$ is the volume, and $T$ is the temperature of humid air. Substituting (2.2) into (2.1) and taking into account that the atmospheric pressure $P_{atm}$ is the sum of the partial pressures of the dry air and the water vapor, we obtain

$$h = \frac{M_W P_W}{M_{DA} P_{DA}} = \frac{M_W P_W}{M_{DA}(P_{atm} - P_{H_2O})} \approx \frac{M_W P_W}{M_{DA} P_{atm}} \tag{2.3}$$

The last equality in (2.3) follows from the fact that $P_W \ll P_{atm}$. We can use a similar approximation to relate the mass fraction $y$ of water vapor in air to humidity,

$$y = \frac{m_W}{m_W + m_{DA}} \approx \frac{m_w}{m_{DA}} = h \tag{2.4}$$

**Saturation humidity** $h_S(T)$ at temperature $T$ is the humidity corresponding to equilibrium between humid air and liquid water at this temperature.

**Relative humidity** is

$$h_R = \frac{P_W}{P_W^{sat}} \cdot 100 \ \%. \tag{2.5}$$

Where $P_W^{sat}$ is the partial pressure of water vapor in saturated air.

**Specific enthalpy** of humid air is

$$H_G \left[ \frac{kJ}{kg \ dry \ air} \right] = H_0 + C_G(T - T_0) + \lambda h, \tag{2.6}$$

where $\lambda$ is the latent heat of water (kJ/kg water vapor), $H_0$ and $T_0$ are the reference enthalpy and temperature, and $C_G$ is the heat capacity of the humid air,

$$C_G \left[ \frac{kJ}{(kg \ dry \ air)K} \right] = C_{DA} + C_W h \tag{2.7}$$

Here, $C_{DA} \approx 1.005$ kJ/(kg dry air)·K and $C_W \approx 1.88$ kJ/(kg water vapor)·K are the heat capacities of dry air and water vapor.

The thermodynamic properties of humid air are summarized in the chart shown in Figure 2-1.

**Wet bulb temperature** $T_{wb}(\underline{T},h)$ of air at temperature $T$ and humidity $h$ is the temperature of saturated air that would be obtained by an adiabatic evaporation of water initially at the same temperature $T$ as the air. If the water to be evaporated is already at the same temperature as the air, all it needs for evaporation is the supply of latent heat, which is taken from the air by reduction of its temperature. Wet bulb temperature of air at the inlet of the cooling tower represents the lowest temperature (**cooling limit**) that can be achieved due to evaporation.

The wet bulb temperature can be obtained from the thermodynamic chart in Figure 2-1 by following the constant-enthalpy line starting at the given $T$ and $h$ until the saturation line. For example, the wet bulb temperature of air at $T = 17°C$ and humidity $h = 6.2$ g/kg is $T_{wb} = 11.2°C$.
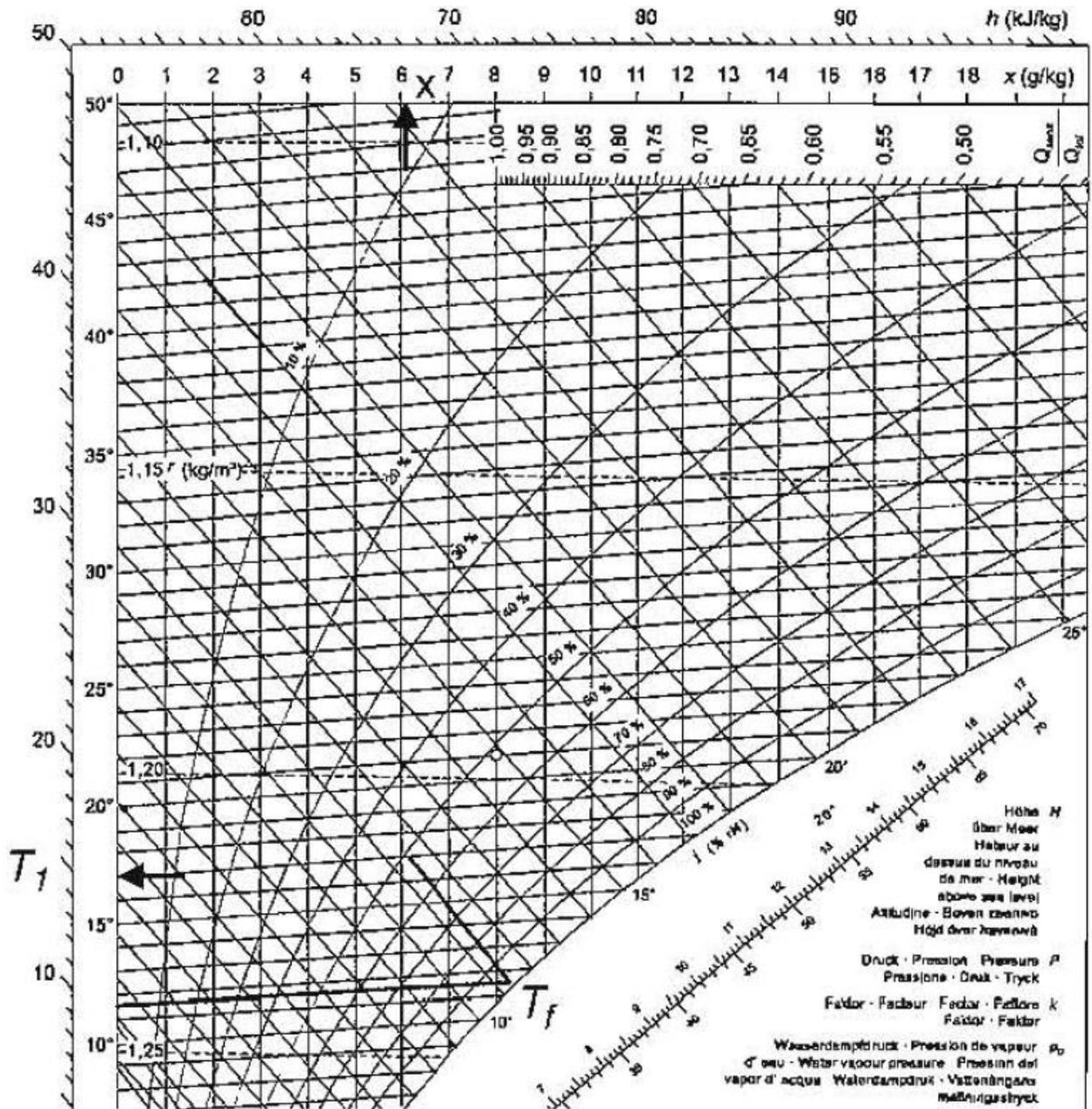
Figure 2-1. Thermodynamic properties of humid air.

## Characteristic Variables of a Cooling Tower

**Cooling Limit** is the wet bulb temperature of ambient air

**Wet Bulb Approach** is defined as

$$a = T_{L,out} - T_{wb}, \tag{2.8}$$

where $T_{L,out}$ is the water temperature at the cooling tower outlet and $T_{wb}$ is the wet bulb temperature.

Each cooling tower has its own wet bulb approach. For example under certain climatic conditions the minimum wet bulb approach for open cooling towers in industrial applications could be between 3 and 5 K. The final temperature of water leaving the cooling tower depends on the temperature of the cooling medium (air) and the design of the cooling tower.

**Cooling Range** $z$ is the difference between the warm water temperature at the inlet to the cooling tower and the cold water temperature at the outlet from the cooling tower,

$$\zeta = T_{L,in} - T_{L,out}, \tag{2.9}$$

where $T_{L,in}$ is the inlet water temperature.

**Cooling Coefficient** is defined as

$$\eta = \frac{T_{L,in} - T_{L,out}}{T_{L,in} - T_{wb}} \tag{2.10}$$

**Water Loss** due to evaporation,

$$\Delta \dot{L} = \dot{m}_G (h_{out} - h_{in}), \tag{2.11}$$

here $\dot{m}_G$ is the mass flow rate of air and $h_{out}$ and $h_{in}$ are the absolute humidities of air at the inlet and outlet, respectively.

**Heat Load** is the rate of energy transfer from water to air,

$$\dot{Q}_W = \dot{m}_L C_L (T_{L,in} - T_{L,out}) \tag{2.12}$$

where $\dot{m}_L$ and $C_L$ are the mass flow rate and the heat capacity of liquid water, respectively.

**Packing Density** is the ratio of the surface areas of all levels of the cooling column to the column volume. Packing density is measured in the units of $m^2/m^3$.

## Material and Energy Flows

The material and energy flows at the interface are summarized in Figure 2-2.

**Material flow:**

Water vapor diffuses from the interface to the bulk gas phase because humidity at in bulk air is lower than the interface. The driving force for this diffusion is $(h_I - h_G)$.

**Energy flows:**

1. **Water phase:** Sensible heat flows from the bulk liquid to the water-air interface; the driving force for this thermal transfer is $T_L - T_I$
2. **Gas Phase:**
   a. Sensible heat flows from the interface to the bulk gas phase; the driving force for this thermal transfer is $T_I - T_G$.
   b. Latent heat of evaporation flows from the interface to the bulk phase.

The sensible heat flowing from liquid to the interface equals the sensible heat flow in the gas plus the latent heat flow in the gas. Note that the direction of the sensible heat flow in gas changes along the column: in the upper part of the column, the interface temperature is higher than that of air and, hence, heat flows from the interface to air. In the lower part of the column, the air temperature may be higher than the temperature at the interface. Hence, the sensible heat flows from air to the interface. However, the latent heat still flows from the interface to the bulk gas phase, thus removing heat from water.
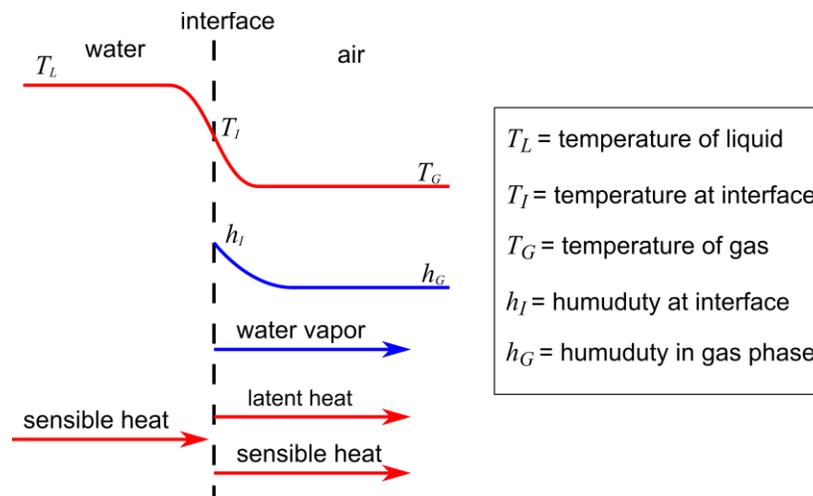


Figure 2-2. Temperature and concentration profiles at the air-water interface in a cooling tower. The energy and material flows across the interface are shown by the blue and red arrows, respectively.

## Analysis of Cooling Towers

Schematics of a cooling tower is shown Figure 2-3. Assume that the rate of water loss is much smaller than the water flow rate, i.e. *L≈const.* Furthermore, we assume that the water heat capacity $C_L$ remains constant throughout the column.
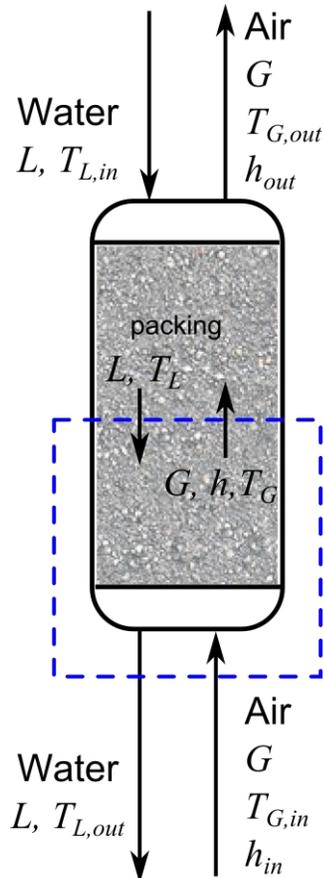


$G$ (kg dry air/s·m²) = air flow rate per unit area

$L$ (kg/s·m²) = flow rate of liquid water per unit area

$H_G$ (J/kg dry air) = specific enthalpy of wet air

$H_L$ (J/kg) = specific enthalpy of liquid water

$T_L$ (K) = temperature of liquid water

$C_L$ (J/kg·K) = heat capacity of liquid water

Figure 2-3. Schematics of a cooling tower

Consider energy balance for a small volume of wet air in the cooling tower:

$$G[H_G(z + dz) - H_G(z)] = k_G a[H_{I,G}(z) - H_G(z)]dz. \tag{2.13}$$

Here, $H_{I,G}$ is the specific enthalpy of wet air at the air-water interface, $k_G a$ (kg/s·m³) is the mass transfer rate per unit volume of packing, and $dz$ is the height of the volume element. The right-hand side of Eq. (2.13) represents the energy transfer due to convection between air-water interfaces and the bulk air. After rearranging and integrating Eq. (2.13), we obtain

$$Z = \int_0^Z dz = \frac{G}{k_G a} \int_{H_{G,in}}^{H_{G,out}} \frac{dH_G(z)}{H_{I,G}(z) - H_G(z)} = h_G n_G \tag{2.14}$$

Here, $Z$ is the height of the packing,

$$h_G = \frac{G}{k_G a} \quad \text{(2.15)}$$

is the **height of a transfer unit** and

$$n_G = \int_{H_{G,in}}^{H_{G,out}} \frac{dH_G}{H_{I,G} - H_G} \quad \text{(2.16)}$$

is the **number of transfer units** in the column. The height of a transfer unit is a measure of effectiveness of the packing in the cooling tower in facilitating mass transfer of water vapor into air. The number of transfer units is a measure of difficulty of mass transfer of water vapor into air for the given operating conditions of the cooling tower.

It is more convenient to use the *overall mass transfer coefficient $k_G a$* instead of the *film mass transfer coefficient $k_G a$*. In this case, the energy balance (2.13) can be rewritten as follows:

$$G[H_G(z + dz) - H_G(z)] = K_G a[H_G^*(z) - H_G(z)]dz, \quad \text{(2.17)}$$

where $H_G^*(z)$ is the enthalpy of the saturated air that would be in equilibrium with water at height $z$ within the column. Using Eq. (2.17), we obtain the following expression for the height of the packing:

$$Z = h_{OG} n_{OG}, \quad \text{(2.18)}$$

where

$$h_{OG} = \frac{G}{K_G a} \quad \text{(2.19)}$$

is the height of a transfer unit and

$$n_{OG} = \int_{H_{G,in}}^{H_{G,out}} \frac{dH_G}{H_G^* - H_G} \quad \text{(2.20)}$$

is the number of transfer units based on the overall mass transfer coefficient.

The integration in (2.20) can be performed numerically, e.g., using the trapezoidal rule:

$$\int_{H_{G,in}}^{H_{G,out}} f(H_G)dH_G = \sum_{k=1}^{N-1} \left[ \frac{f(H_{G,k}) + f(H_{G,k+1})}{2} \right] \Delta H_G, \quad \text{(2.21)}$$

where

$$f(H_G) = \frac{1}{H_G^*(H_G) - H_G},$$

(2.22)

$$\Delta H_G = \frac{H_{G,out} - H_{G,in}}{N - 1},$$

(2.23)

and

$$H_{G,k} = H_{G,in} + (k - 1)\Delta H_G$$

(2.24)

In particular,

$$H_{G,1} = H_{G,in} \text{ and } H_{G,N} = H_{G,out}$$

(2.25)

To perform the integration (2.21), we need to obtain the relationship between $H_G(z)$ and $H_G^*(z)$. To do this, we need to obtain the liquid temperature $T_L(z)$ at height $z$ of the column. This can be done using the energy balance for the area inside the dashed rectangle in Figure 2-3:

$$G[H_{G,in} - H_G(z)] = C_L L[T_{L,out} - T_L(z)]$$

(2.26)

Once the liquid temperature $T_L(z)$ is known, we can obtain enthalpy $H_G^*(z)$ of saturated (100% humidity) air at this temperature either from the psychrometric chart or from Eqs. (2.6), (2.7)[1].

This will allow us to generate a table similar to the one below:

| $H_G(z)$ | $T_L(z)$ | $H_G^*(z)$ |
|---|---|---|
| $H_{G,1} = H_{G,in}$ | $T_{L,1} = T_{L,out}$ | $H_{G,1}^* = H_G^*(T_{L,1})$ |
| $H_{G,2} = H_{G,in} + \Delta H_G$ | $T_{L,2}$ | $H_{G,2}^* = H_G^*(T_{L,2})$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $H_{G,N} = H_{G,out}$ | $T_{L,in}$ | $H_{G,N}^* = H_G^*(T_{L,N})$ |

This table provides the relationship between $H_G$ and $H_G^*$, which can be used to obtain $f(H_G)$ and perform the integration (2.21).

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[1] Recall that $h$ in Eqs. (2.6), (2.7) is the *absolute* humidity.

## Empirical Correlations for Mass Transfer Coefficient

After the values of $K_Ga$ are obtained experimentally for various air and water flow rates, the dependence of $K_Ga$ on the flow rates can be summarized using the following empirical correlation:

$$K_G a = C_1 G^{C_2} L^{C_3}. \tag{2.27}$$

The values of constants $C_1$, $C_2$, and $C_3$ can be obtained by taking the natural logarithm of the power law (2.27),

$$\ln(K_G a) = \ln C_1 + C_2 \ln G + C_3 \ln L, \tag{2.28}$$

and applying the linear regression analysis to the obtained Eq. (2.28). For example, $C_3$ corresponds to the slope of $\ln(K_Ga)$ vs. $\ln L$ plotted for experimental data corresponding to the same $G$ but different $L$, as illustrated in Figure 2-4. Similarly, $C_2$ can be obtained from the slope of $\ln(K_Ga)$ vs. $\ln G$ corresponding to the data with the same $L$. $C_1$ can be obtained from the average intercept of these two lines.
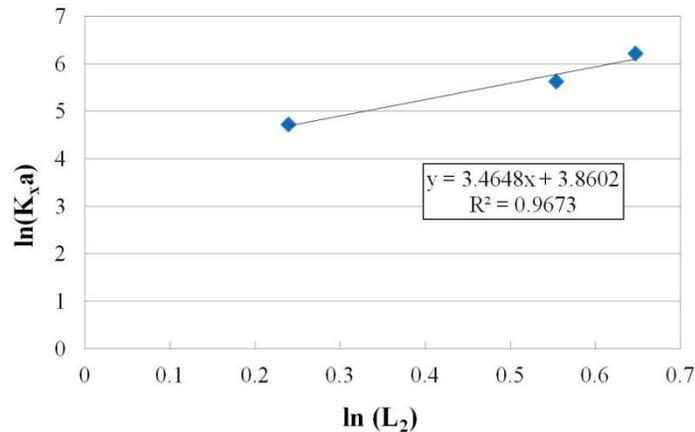


Figure 2-4. Example of log-log plot of $K_Ga$ versus $L$ at constant $G$.